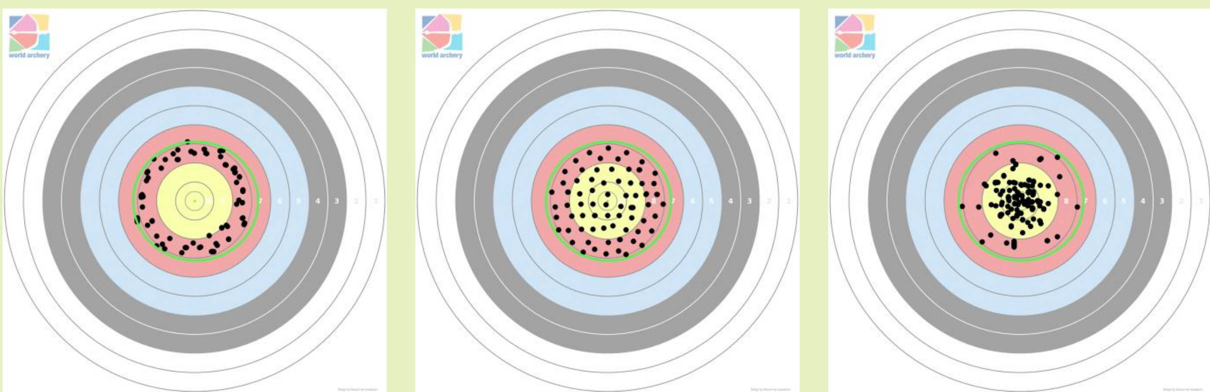
Archer’s Skill Level

Most “green” articles on ***ArtemisLite*** are short and easy to understand. This article is different as it tries to explain the theoretical concept of the Archer’s Skill Level (ASL, or in Dutch; “Intrinstiek Scorend Vermogen” or ISV) and the somewhat difficult mathematical concept upon which it is build.

Arrow grouping

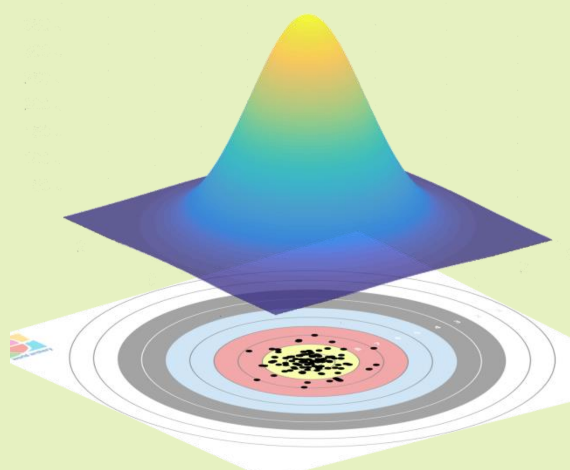
Let’s start with the archer shooting its arrows at a target face with circular rings. As you would expect, the inner circle has the highest point value. The archer shoots several arrows in one end (and in some competitions only one arrow per end) and does this a number of ends. Without adjusting the sight, the arrows shot will form a group.



Same group size, different kinds of grouping patterns

It is a common mistake to think that the group *size* is the only thing important for the score or performance of the archer. In the pictures above, three groups with the same *size* are shown. Clearly the group on the right is to be preferred from a scoring point of view. It shows that what defines the performance of an archer is the average group size ***and*** distribution of arrows within that group.

Making a cross-section of the group, most archers groups will look like the figure on the right. Such a figure is called the distribution density function.



The standard distribution

Under certain assumptions (normally distributed groups), these groups can be characterized by a single parameter; the group standard deviation σ

Let's start with some advanced math; a random variable *X* is said to be normally distribute with mean μ_X and variance σ_X^2 if its probability density function is;

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_X} e^{\left(-\frac{(x-\mu_X)^2}{2\sigma_X^2}\right)}$$

If the position *P* of a single arrow in a right-handed coordinate reference system is denoted *(x,y)*.

And if we assume;

- The group is normally distributed.
- The archers sight is set correct, thus the average of the group will be at the center *(0,0)*, thus $\mu = \mu_X = \mu_Y = 0$.
- The group is circular, which means the standard deviation in *x*-direction equals that of the *y*-direction, $\sigma = \sigma_X = \sigma_Y$.
- The *x*- and *y*-component are independent.

Later in this article we will validate these assumptions.

The distribution density function for the 2 dimensional *(x,y)* case then becomes;

$$f_{X,Y}(x,y) = \frac{1}{\sqrt{2\pi}\sigma_X} e^{\left(-\frac{(x-\mu_X)^2}{2\sigma_X^2}\right)} \cdot \frac{1}{\sqrt{2\pi}\sigma_Y} e^{\left(-\frac{(y-\mu_Y)^2}{2\sigma_Y^2}\right)}$$

With $\mu = \mu_X = \mu_Y = 0$ and $\sigma = \sigma_X = \sigma_Y$, this becomes

$$f_{X,Y}(x,y) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^2 \cdot \left[e^{\left(-\frac{(x)^2}{2\sigma^2}\right)} \cdot e^{\left(-\frac{(x)^2}{2\sigma^2}\right)}\right]$$

$$f(x,y) = \frac{1}{2\pi\sigma^2} e^{\left(-\frac{1}{2}\left[\left(\frac{x}{\sigma}\right)^2 + \left(\frac{y}{\sigma}\right)^2\right]\right)}$$

Changing to polar coordinates, with $r^2 = x^2 + y^2$ and $\varphi = \arctan\left({}^{y}/_{x}\right)$ gives the distribution density function as a function of radius and angle.

$$f(r,\varphi) = \frac{1}{2\pi\sigma^2} e^{\left(-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2\right)}$$

Integrating this distribution density function over 2π and from 0 to r gives the probability density function and a function of the radius *r*

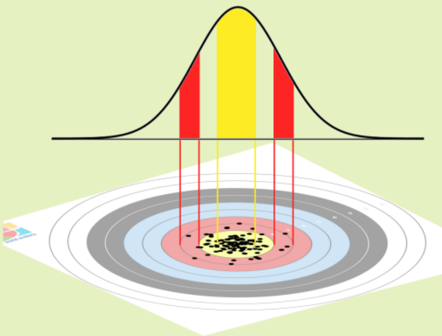
$$f(r) = \int_0^{2\pi} \int_0^{r} f(r,\varphi) r \, dr \, d\varphi = \frac{r}{\sigma^2} e^{\left(-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2\right)}$$

The cumulative probability function *F(r)* is again integrating from 0 to radius *r*

$$F(x) = \int_0^{x} f(r) dr \ \rightarrow F(r) = 1 - e^{\left(-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2\right)}$$

This function calculates the probability of an arrow landing within distance *r* of the center for a given value of σ. The value of σ represents the skill level of an archer.

The expected score for an arrow, *S*, is then determined by overlaying the probability distribution on the target face rings and multiplying each rings probability with the value of that particular ring and then adding the result.



The probability F_i of an arrow with radius R_{arrow} hitting (or having a liner) within a certain target face ring with radius R_i is:

$$F_i = 1 - e^{\left(-\frac{1}{2}\left(\frac{R_i + R_{arrow}}{\sigma}\right)^2\right)}$$

The average arrow value *S* can be computed by calculating each probability contribution (from the outside of a ring *i+1* to the outside of ring *i*) and multiplying this with the value of the ring *i* and then add all contributions for all rings on the target face.

$$S = \sum_{i=1}^{n} \{(F_i - F_{i+1}) \cdot V_i\}$$

F_i is the probability that the arrow hits within or is touching the *i* ring.
n = the number of scoring rings on the target face, ring *i*=1 is the outer target ring and there is no ring higher than *n*. So $F_{n+1} = 0$
V_i is the scoring value of ring *i* on the target face,
R_i is the radius of ring *i* on the target face in mm,
R_{arrow} is the average radius of the arrow shaft in mm.

The total score *T when shooting N* arrows, becomes;

$$T = N \cdot S$$

What we have until now;

If the grouping pattern of arrows behaves like a normally distributed group with standard deviation σ we can (using the described procedure) compute the average score of a single arrow (and thus the total score of N arrows) shot at some circular target face with rings that have certain scoring values

The archer's skill level model

Now that we have a procedure of finding an arrow average, given a standard deviation. The next step is to find a value for this standard deviation.

The whole idea behind finding a model for the archer's shooting performance is that the skill of an archer remains constant; independent of how many arrows are shot, what kind of target-face geometry and scoring rules, and independent of shooting distance. Once the arrow leaves the bow, it doesn't matter how far it flies or what kind of target it encounters on the way. The accuracy of the arrow is all determined by the archer already.

In [1] such a model was proposed and validated. The validation consisted of using actual scoring results of a big population of high level archers shooting different distances in near perfect weather. The model is denoted the (revised) Archery Australia Archer's Skill Level model. It uses a given number *I* that represents the archer's skill level and a distance *D* to compute a standard deviation $\sigma = f(I,D)$

$$\sigma = \left(0.815 \cdot D + 0.185 \cdot \left({D^2}/{50}\right)\right) \cdot e^{(-0.027 \cdot I + 2.57)}$$

Where *D* is the distance to the target in meters and *I* is called the archer's skill level index. We see in this model that the standard deviation (or group size) increases with distance.

In the study mentioned ([1]) each participating archer was assumed to have a constant skill level and the scores of every archer on 4 different distances was compared using this model. It was found that the modeling errors were well below 1 point and that the model was valid.

This means that given a recurve men score on e.g. 90m, the model may be used to calculate the required skill level for shooting such a score. Once we have an indication of the skill level and we assume th at an archer will shoot the next distances with the same skill level, we can actually predict the scores of the archer on the other distances (70m, 50m and 30m).

$$S_1 \leftrightarrow I_{constant} \leftrightarrow S_2$$

In the example below the model is applied;

As an example, let's try to find the average arrow score of an archer shooting with 5mm diameter arrows, with skill level 100 shooting at 122cm target face at 70m.

$$\sigma = \left(0.815 \cdot 70 + 0.185 \cdot \left({70^2}/{50}\right)\right) \cdot e^{(-0.027 \cdot 100 + 2.57)}$$

$$\sigma = 75.18 \cdot e^{-0.13}$$

$$\sigma = 66.015$$

Let's look at the 10-ring, for which *i* = *n*. The radius is 61mm.

$$F_{11} = 0 \text{ (by definition)}$$

$$F_{10} = 1 - e^{\left(-\frac{1}{2}\left(\frac{61+2.5}{66.015}\right)^2\right)} = 0.3704$$

$$F_9 = 1 - e^{\left(-\frac{1}{2}\left(\frac{122+2.5}{66.015}\right)^2\right)} = 0.8311$$

$$F_8 = 1 - e^{\left(-\frac{1}{2}\left(\frac{183+2.5}{66.015}\right)^2\right)} = 0.9807$$

If we put these results in a table, we get

Cumulative probability function		*Ring value*	*Score contribution*	
F_{11}	0	N/a	0	0
F_{10}	0.3704	10	$(F_{10}-F_{11})*V_{10}$	3.7037
F_9	0.8311	9	$(F_9-F_{10})*V_9$	4.1464
F_8	0.9807	8	$(F_8-F_9)*V_8$	1.1970
F_7	0.9991	7	$(F_7-F_8)*V_7$	0.1284
F_6	1.0000	6	$(F_6-F_7)*V_6$	0.0055
$F_5..F_1$	1.0000	5…1	…	0.0001
			Arrow average	***9.1812***

So, an archer with a skill-level of 100 will be able to shoot a 72 arrow-round score on 122cm target face at 70m of 72*9.1812 of **661**

As the example shows, this model is able to find a score from a given archer's skill level, a distance and target face geometry including ring values.

References

[1] *"Analysis of scores and arrow grouping at major international archery competitions"*, James L Park, James E Larven. Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology, vol. 228, 2: pp. 86-94. , First Published January 28, 2014.