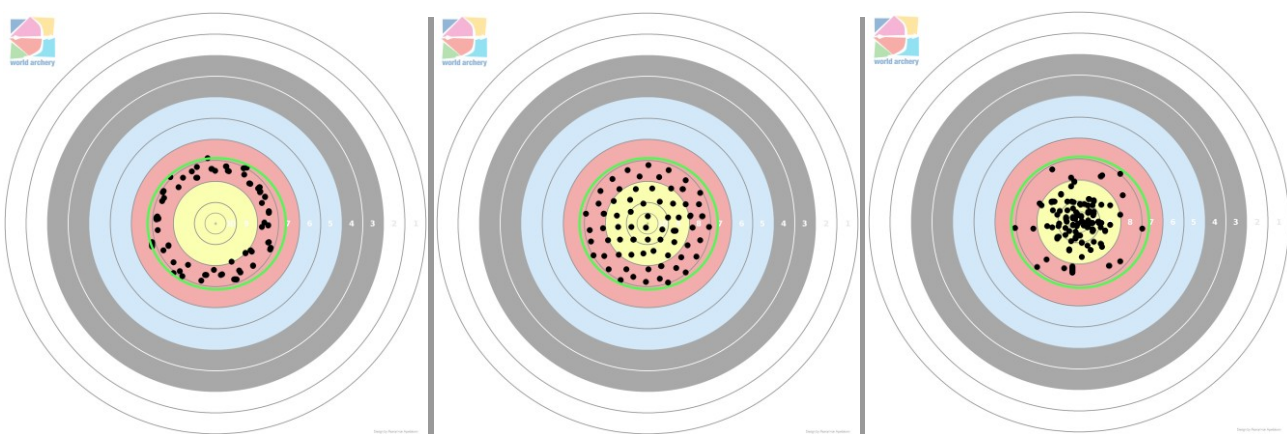


Archer's Skill Level

Most "green" articles on ArtemisLite are short and easy to understand. This article is different as it tries to explain the theoretical concept of the Archer's Skill Level (ASL, or in Dutch; "Intrinstiek Scorend Vermogen" or ISV) and the somewhat difficult mathematical concept upon which it is build.

Arrow grouping

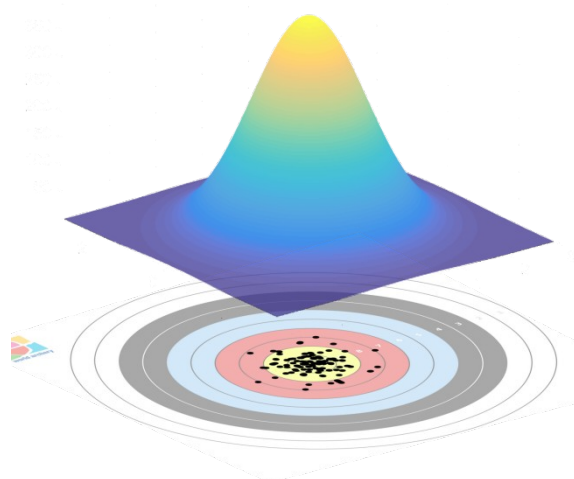
Let's start with the archer shooting its arrows at a target face with circular rings. As you would expect, the inner circle has the highest point value. The archer shoots several arrows in one end (and in some competitions only one arrow per end) and does this a number of ends. Without adjusting the sight, the arrows shot will form a group.



Same group size, different kinds of grouping patterns

It is a common mistake to think that the group size is the only thing important for the score or performance of the archer. In the pictures above, three groups with the same size are shown. Clearly the group on the right is to be preferred from a scoring point of view. It shows that what defines the performance of an archer is the average group size **and** distribution of arrows within that group.

Making a cross-section of the group, most archers groups will look like the figure on the right. Such a figure is called the distribution density function.





The standard distribution

Under certain assumptions (normally distributed groups), these groups can be characterized by a single parameter; the group standard deviation σ

Let's start with some advanced math; a random variable X is said to be normally distributed with mean μ_x and variance σ_x^2 if its probability density function is;

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{\left(\frac{-(x-\mu_x)^2}{2\sigma_x^2}\right)}$$

If the position P of a single arrow in a right-handed coordinate reference system is denoted (x,y) . And if we assume;

- o The group is normally distributed.
- o The archers sight is set correct, thus the average of the group will be at the center $(0,0)$, thus $\mu = \mu_x = \mu_y = 0$.
- o The group is circular, which means the standard deviation in x -direction equals that of the y -direction, $\sigma = \sigma_x = \sigma_y$.
- o The x - and y -component are independent.

Later in this article we will validate these assumptions. The distribution density function for the 2 dimensional (x,y) case then becomes;

$$f_{X,Y}(x,y) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{\left(\frac{-(x-\mu_x)^2}{2\sigma_x^2}\right)} \cdot \frac{1}{\sqrt{2\pi}\sigma_y} e^{\left(\frac{-(y-\mu_y)^2}{2\sigma_y^2}\right)}$$

With $\mu = \mu_x = \mu_y = 0$ and $\sigma = \sigma_x = \sigma_y$, this becomes

$$f_{X,Y}(x,y) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^2 \cdot \left[e^{\left(\frac{-(x)^2}{2\sigma^2}\right)} \cdot e^{\left(\frac{-(y)^2}{2\sigma^2}\right)} \right]$$

$$f(x,y) = \frac{1}{2\pi\sigma^2} e^{\left(\frac{-1}{2}\left[\left(\frac{x}{\sigma}\right)^2 + \left(\frac{y}{\sigma}\right)^2\right]\right)}$$

Changing to polar coordinates, with $r^2 = x^2 + y^2$ and $\varphi = \arctan\left(\frac{y}{x}\right)$ gives the distribution density function as a function of radius and angle.

$$f(r,\varphi) = \frac{1}{2\pi\sigma^2} e^{\left(\frac{-1}{2}\left(\frac{r}{\sigma}\right)^2\right)}$$

Integrating this distribution density function over 2π and from 0 to r gives the probability density function and a function of the radius r



$$f(r) = \int_0^{2\pi} \int_0^r f(r, \varphi) r dr d\varphi = \frac{r}{\sigma^2} e^{-\frac{1}{2} \left(\frac{r}{\sigma} \right)^2}$$

The cumulative probability function $F(r)$ is again integrating from 0 to radius r

$$F(x) = \int_0^x f(r) dr \rightarrow F(r) = 1 - e^{-\frac{1}{2} \left(\frac{r}{\sigma} \right)^2}$$

This function calculates the probability of an arrow landing within distance r of the center for a given value of σ . The value of σ represents the skill level of an archer.

The expected score for an arrow, S , is then determined by overlaying the probability distribution on the target face rings and multiplying each rings probability with the value of that particular ring and then adding the result.

The probability F_i of an arrow with radius R_{arrow} hitting (or having a liner) within a certain target face ring with radius R_i is:

$$F_i = 1 - e^{-\frac{1}{2} \left(\frac{R_i + R_{arrow}}{\sigma} \right)^2}$$

The average arrow value S can be computed by calculating each probability contribution (from the outside of a ring $i+1$ to the outside of ring i) and multiplying this with the value of the ring i and then add all contributions for all rings on the target face.

$$S = \sum_{i=1}^n \left\{ (F_i - F_{i+1}) \cdot V_i \right\}$$

F_i = is the probability that the arrow hits within or is touching the i ring.

n = the number of scoring rings on the target face, ring $i=1$ is the outer target ring and there is no ring higher than n . So $F_{n+1} = 0$

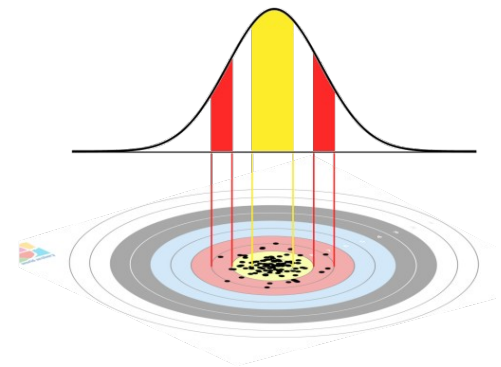
V_i = the scoring value of ring i on the target face,

R_i = the radius of ring i on the target face in mm,

R_{arrow} = the average radius of the arrow shaft in mm.

The total score T when shooting N arrows, becomes;

$$T = N \cdot S$$





What we have until now;

If the grouping pattern of arrows behaves like a normally distributed group with standard deviation σ we can (using the described procedure) compute the average score of a single arrow (and thus the total score of N arrows) shot at some circular target face with rings that have certain scoring values

The archer's skill level model

Now that we have a procedure of finding an arrow average, given a standard deviation. The next step is to find a value for this standard deviation.

The whole idea behind finding a model for the archer's shooting performance is that the skill of an archer remains constant; independent of how many arrows are shot, what kind of target-face geometry and scoring rules, and independent of shooting distance. Once the arrow leaves the bow, it doesn't matter how far it flies or what kind of target it encounters on the way. The accuracy of the arrow is all determined by the archer already.

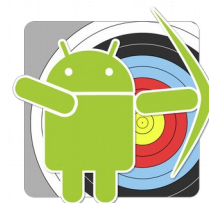
In [1] such a model was proposed and also validated. The validation consisted of using actual scoring results of a big population of high level archers shooting different distances in near perfect weather. The model is denoted the (revised) Archery Australia Archer's Skill Level model. It uses a given number I that represents the archer's skill level and a distance D to compute a standard deviation $\sigma = f(I, D)$

$$\sigma = \left(0.815 \cdot D + 0.185 \cdot \left(\frac{D^2}{50} \right) \right) \cdot e^{(-0.027 \cdot I + 2.57)}$$

Where D is the distance to the target in meters and I is called the archer's skill level index. We see in this model that the standard deviation (or group size) increases with distance. In the study mentioned ([1]) each participating archer was assumed to have a constant skill level and the scores of every archer on 4 different distances was compared using this model. It was found that the modeling errors were well below 1 point and that the model was valid.

This means that given a score S_1 on one distance and target face, the model may be used to calculate the required skill level for shooting such a score. Once we have an indication of the skill level and we assume that an archer will shoot the next distances with the same skill level, we can actually predict the score of the archer on another distance and/or target face S_2 .

$$S_1 \leftrightarrow I_{constant} \leftrightarrow S_2$$



An example;

As an example, let's try to find the average arrow score of an archer shooting with **5mm diameter arrows**, with **skill level 100** shooting at **122cm** target face at **70m**.

$$\sigma = \left(0.815 \cdot 70 + 0.185 \cdot \left(\frac{70^2}{50} \right) \right) \cdot e^{(-0.027 \cdot 100 + 2.57)}$$

$$\sigma = 75.18 \cdot e^{-0.13}$$

$$\sigma = 66.015$$

Let's look at the 10-ring, for which $i = n$. The radius is 61mm.

$$F_{11} = 0 \text{ (by definition)}$$

$$F_{10} = 1 - e^{\left(\frac{-1 \left(\frac{61+2.5}{66.015} \right)^2}{2} \right)} = 0.3704$$

$$F_9 = 1 - e^{\left(\frac{-1 \left(\frac{122+2.5}{66.015} \right)^2}{2} \right)} = 0.8311$$

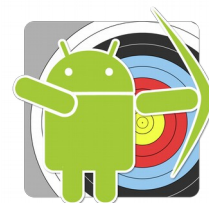
$$F_8 = 1 - e^{\left(\frac{-1 \left(\frac{183+2.5}{66.015} \right)^2}{2} \right)} = 0.9807$$

If we put these results in a table, we get

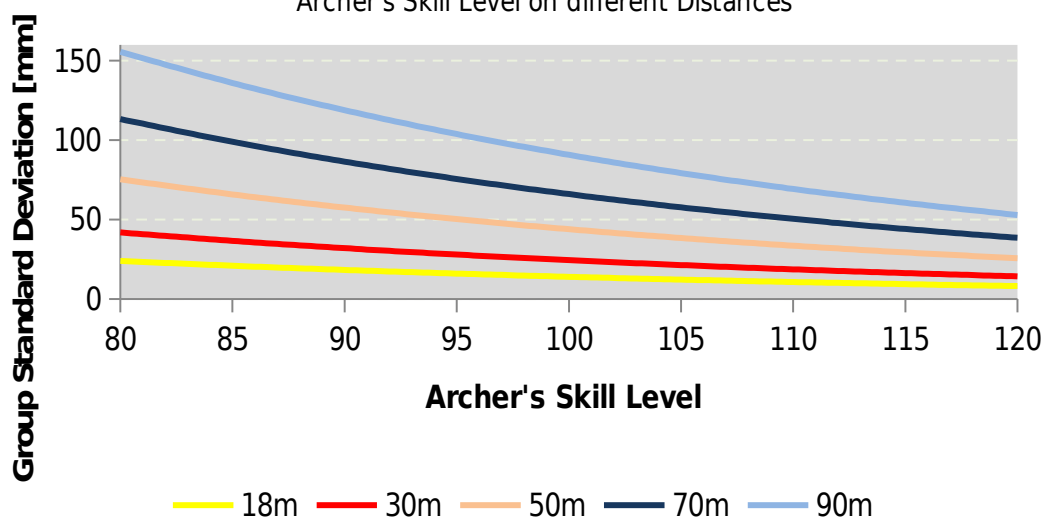
Cumulative probability function		Ring value	Score contribution	
F_{11}	0	n/a	n/a	n/a
F_{10}	0.3704	10	$(F_{10}-F_{11}) \cdot V_{10}$	3.7037
F_9	0.8311	9	$(F_9-F_{10}) \cdot V_9$	4.1464
F_8	0.9807	8	$(F_8-F_9) \cdot V_8$	1.1970
F_7	0.9991	7	$(F_7-F_8) \cdot V_7$	0.1284
F_6	1.0000	6	$(F_6-F_7) \cdot V_6$	0.0055
$F_{5..F_1}$	1.0000	5..1	...	0.0001
			Arrow average	9.1812

So, an archer with a skill-level of 100 will be able to shoot a 72 arrow-round score on 122cm target face at 70m of $72 \cdot 9.1812$ of **661**

As the example shows, this model is able to find a score from a given archer's skill level, a distance and target face geometry including ring values.



Group Standard Deviation for given Archer's Skill Level on different Distances



In the graph, the group standard deviation (in [mm]) given an archer's skill level for different distances is shown. For increasing skill levels, the group size decreases. Also for decreasing distance, the group size decreases as well.

How can we use the Archer's Skill Level from a coaching perspective?

For example; these are actual scores of a recurve woman shooting a 1440 round: 70m 311, 60m 324, 50m 308 and 30m 326 for a total of 1269. Just by looking at these scores, it is kind of hard to find the odd-one out (the score where you as coach could ask the athlete "what happened?")

	70m	60m	50m	30m	Total
Score	311	324	308	326	1269
ASL	87.5	88.5	86.0	75.0	85.0

Looking at the equivalent Archer's Skill Levels of each score, it is easy to see that the skill level at 30m was significantly lower than the rest of the round. Given an average skill level of around 85, the 30m score should have been around 339 or more. So, something happened, but what? That's your job as a coach to find out.

References

[1] "Analysis of scores and arrow grouping at major international archery competitions", James L Park, James E Larven. Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology, vol. 228, 2: pp. 86-94. , First Published January 28, 2014.