Effects of Bow Lean or Cant on Archers Score

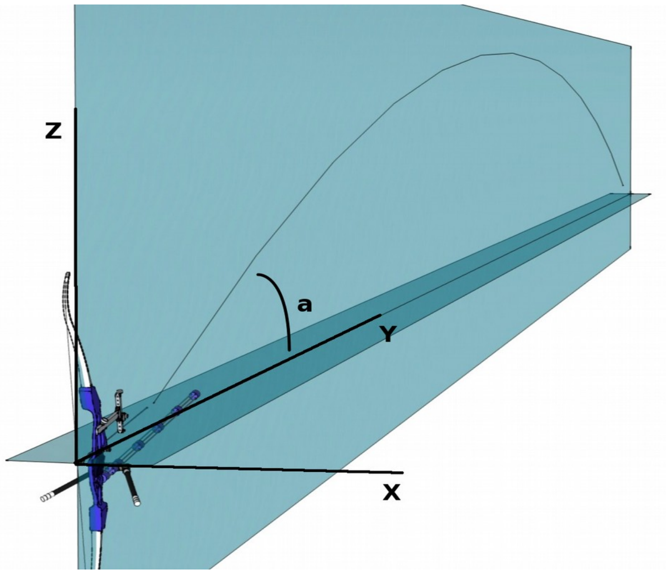
In the compound discpline, bow lean or cant is well known. The bubble-level in the scope is also quite "in your face", making it hard to not be aware of keeping your bow vertical. In recurve however, it is less obvious, but it is in recurve where variations in bow lean or canting is extremely important.

This article belongs to a series of Articles that were written during the development of Artemis (an Android app, used in high-level archery). The article starts with a mathematical derivation of the effect of bow cant on the arrows position on the target. If you're not into math, just skip this section

Bow cant; Beware of the Math section

In the ideal situation where an arrow is shot from a bow that was kept perfectly vertical, the arrow velocity will only have a horizontal (towards the target) and a vertical (opposite to gravity vector) component.

In a coordinate reference system where the positive Z-axis is pointing vertical upwards (exactly opposite to the gravity vector), the positive Y-axis points to the target and the positive X-axis completes a right handed reference system, the trajectory lies in the Y-Z plane and is launched with an angle of α[1]



This is how we usually think of arrow trajectories;

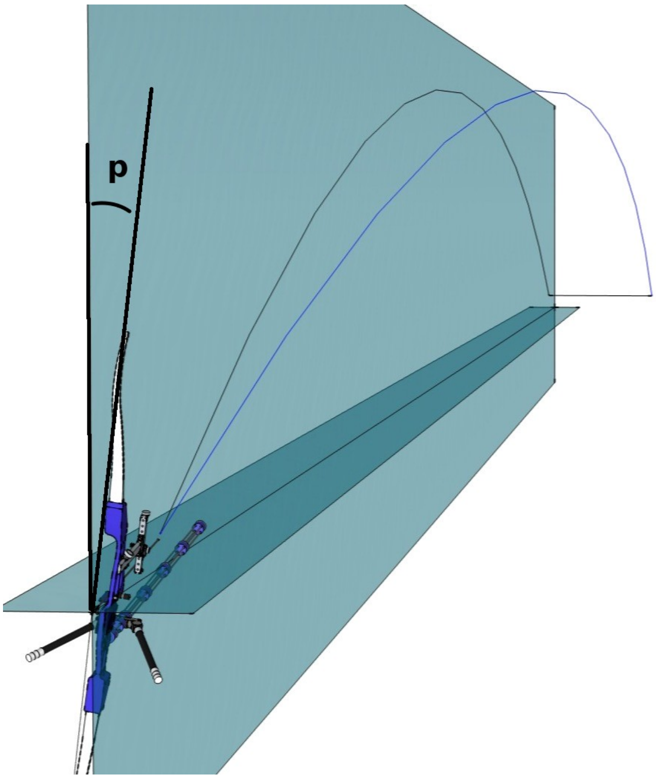
The magnitude of the initial velocity vector is defined by the energy put in by the bow and is constant;

$$\|V\| = constant$$

The launch angle is denoted α which gives horizontal and a vertical speed components;

$$V_Y = \|V\| \cos(\alpha)$$
$$V_Z = \|V\| \sin(\alpha)$$

1 α is 'a' in the picture



Bow lean or cant can be defined as the angle ϕ between the bow's vertical plane and the true vertical (defined by gravity) or Z-axis. See picture 2[2]. Bow canting introduces an horizontal offset (and a minor vertical offset) on the target. The horizontal offset can be quite significant as will be shown here.

We can express the initial velocity components as (note that the velocity magnitude, governed by the energy in the bow, does not change). Because we rotate the entire system from picture 1 around the Y-axis, the velocity in Y direction remains the same;

$$V_Y = \|V\| \cos(\alpha)$$

Velocity in both X and Z-direction change with cant angle ϕ

$$V_Z = \|V\| \sin(\alpha) \cos(\phi)$$

$$V_X = \|V\| \sin(\alpha) \sin(\phi)$$

Due to all kinds of aerodynamic effects, the velocity will not be constant throughout the trajectory. But if we assume an average velocity of $\|V_{avg}\|$ and a parabolic flight trajectory, the total flight time (the time it takes the arrow from initial launch to impact on the target) can be expressed as;

$$T_{flight} = \frac{2\|V_{avg}\| \sin(\alpha) \cos(\phi)}{g}$$

We also know the distance to the target D_{target} and using it we can express the flight time as well;

$$D_{target} = T_{flight} \|V_{avg}\| \cos(\alpha)$$

$$T_{flight} = \frac{D_{target}}{\|V_{avg}\| \cos(\alpha)}$$

2 ϕ is 'p' in the picture

Combining gives;

$$\frac{2\|V_{avg}\|\sin(\alpha)\cos(\phi)}{g}=\frac{D_{target}}{\|V_{avg}\|\cos(\alpha)}$$

$$2(\|V_{avg}\|)^2\sin(\alpha)\cos(\alpha)\cos(\phi)=g\,D_{target}$$

$$2\sin(\alpha)\cos(\alpha)=\frac{g\,D_{target}}{\cos(\phi)(\|V_{avg}\|)^2}$$

$$\sin(2\alpha)=\frac{g\,D_{target}}{\cos(\phi)(\|V_{avg}\|)^2}$$

$$\boxed{\alpha=\frac{1}{2}\arcsin\left(\frac{g\,D_{target}}{\cos(\phi)(\|V_{avg}\|)^2}\right)}$$

The horizontal offset $D_{offset,X}$ on the target due to the average velocity component V_X is;

$$D_{offset,X}=V_X.T_{flight}$$

$$D_{offset,X}=\|V_{avg}\|\sin(\alpha)\sin(\phi).\frac{D_{target}}{\|V_{avg}\|\cos(\alpha)}$$

$$D_{offset,X}=\frac{\sin(\alpha)}{\cos(\alpha)}\sin(\phi)D_{target}$$

$$\boxed{D_{offset,X}=\tan(\alpha)\sin(\phi)D_{target}}$$

The real-world effects

Now that we have done the math, let's look at some real-world applications. What are the effects? Suppose we have a bow cant angle of 1 degree. For a typical recurve target bow this means that the tip of the limb of the bow (at full draw) is moved about 14mm out of plane. Such an angle would lead on 70m with a typical average arrow speed of 55m/s[3] to an offset of 140mm on the target. 140mm means an '8' instead of a dead-center 'X'.

We know that if the archer is very consistent, the sight can be adjusted for this offset and an 'X' can still be scored every time. Therefore bow canting in itself is not a major issue, it is the *variation* in bow cant that needs to be considered.

However, being so close to vertical, the numerical example is still valid. If a recurve archer *varies* 1 degree, the offset on the target at 70m is still 140mm.

For compound, due to the shorter distance (50m) and the higher average arrow speeds, the numbers are a bit different. 1 degree variation from normal – for a typical compoundbow this would mean the top cam moves ~8mm sideways – with typical average arrow speed of 79m/s results in 34mm offset on the target. Still a 10, but only just...

3 Recurve arrows usually leave the bow with a higher speed (~60..65m/s) but decelerate quite a bit during their flight to 70m. The 55m/s is a pretty accurate average speed.